2016 IEEE Conference on Control Applications (CCA)
Part of 2016 IEEE Multi-Conference on Systems and Control
September 19-22, 2016. Buenos Aires, Argentina

Flight Path and Altitude Tracking Control Of An Impaired Nonlinear Generic Transport Model (GTM) Aircraft With Elevator Jam Failures*

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Abstract— **In this paper, a challenging control system design problem is considered to mitigate the aircraft safety threat posted by elevator jam anywhere within the feasible level flight trim region. A new fixed linear servomechanism-based tracking controller with a nonlinear parameter adaptation was designed to achieve stability, maintain a level flight, and perform accurate altitude tracking for an impaired GTM aircraft with elevator jam failure at any position within the feasible trim region.**

I. INTRODUCTION

According to a recent statistical report [1], in-flight loss-of-control has been the leading cause of fatal airliner aircraft accidents in the past twenty years, and most of the in-flight loss-of-control accidents were triggered by the precursor events including subsystem/component failures, external hazards, and human errors [2], [3], [4].

For the purpose of investigating the flight dynamics and behavior of the civil transport jet aircraft under adverse flight conditions and searching for means to prevent in-flight loss-of-control accidents, NASA has built generic transport model (GTM) aircraft [5], a twin-turbine unmanned aerial vehicle (UAV), as a test bed. The UAV is 5.5% dynamically scaled to realistically simulate characteristics of a full-scale large civil transport jet aircraft.

The particular adverse precursor event to be considered in this paper is the elevator jam during flight. If the elevator is stuck at a non-neutral position during flight, it will post immediate threat to the aircraft and may lead to a crash. In this paper, we will investigate how an elevator jam would affect the flight of the GTM aircraft and present a feasible and effective approach to mitigate the effect of the elevator failure, stabilize the flight, and control the flight path/altitude of the impaired aircraft.

To better understand how serious a threat an elevator jam can post to an aircraft, one can read the account of the NTSB aircraft accident report [6] on the crash of Alaska Airlines Flight 261. The first fault the Flight 261 crew members encountered was a horizontal stabilizer jam at $0.4°$, which was near the neutral trim condition. This fault was not severe and the pilots were able to keep the aircraft aloft at 31,050 feet. But about twenty minutes later, the horizontal stabilizer was moved by an excessive force from $0.4°$ to a new jam position, $2.5°$ airplane nose down, and the airplane began to pitch nose down, starting a dive. Then the pilots lost control of the vertical pitch, and the aircraft crashed into the ocean 11 minutes and 37 seconds later.

An early approach to accommodate the actuator jam failure was the Pseudo-inverse Method or the Mixer Approach [7], [8], [9], [10]. This approach proposed to redesign the controller for the system with actuator failure so that the redesigned closed-loop characteristic matrix would approximate that of the original closed-loop system. In [11], Gao and Antsaklis pointed out that the Pseudo-inverse Method does not guarantee the stability of the redesigned closed-loop system. Consequently, they proposed a modified pseudo-inverse algorithm to resolve the stability issue. In [12], Tao, Joshi, and Ma proposed an adaptive state feedback control approach to achieve plant-model state matching in the presence of actuator failures. Some actuators were assumed to fail and jam at any fixed position during the operation. All of the above approaches assumed that the system dynamics before and after actuator jam are described by the same structure of linear state equations referring to the same equilibrium, and the effect of the actuator failures only changes the system matrices of the linear state equations. In fact, actuator jam will change the equilibrium unless the jam position is at the neutral position. Therefore, stabilizing the impaired system at the original equilibrium actually does not make sense since the equilibrium of the nominal system has disappeared after the actuator jam at a non-neutral position.

To accommodate various elevator jam positions by one single fixed controller, we proposed a servomechanism-based approach to successfully mitigate the effect caused by unknown constant jam positions for linear plants [13]. In [14], the approach was employed to design altitude tracking control system for linearized longitudinal models of the GTM aircraft. But the tracking inaccuracy issue caused by the dynamic nonlinearities remained unsolved until our recent work in [15]. We developed a new approach to design a fixed controller that can stabilize the impaired aircraft and automatically lead the aircraft to a level flight equilibrium determined by the arbitrary elevator jam position within the feasible level flight trim region. This new idea is further developed in this paper to resolve the tracking inaccuracy due to the trim difference before and after the jam failure and

*This work was supported in part by Army Research Laboratory under contract W911NF-15-2-0042.

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extend the range of jam positions that can be covered by one fixed controller. The proposed approach now only required two fixed linear controllers to cover the range of the elevator jam positions from -4 degree to 4 degree to effectively stabilize the system, guarantee level flight at steady state, and achieve accurate altitude tracking.

The remainder of the paper is organized as follows. In Section II, we briefly review the nomenclature of the GTM aircraft model, the level flight trim graphs and the flight simulation diagram using the full nonlinear untrimmed GTM model with practically constrained actuators. Section III provides the controller design procedures for the Accommodation Controllers that are capable of mitigate the effect caused by the elevator jam at any position between -4 and 4 degrees. This elevator jam range from -4 to 4 degrees approximately matches the feasible level flight trim region. Outside the range, there either exists no level flight trim or the trim is close to a stall flight condition. In Section IV, we demonstrate the effectiveness of the proposed actuator jam accommodation approach for a full untrimmed nonlinear GTM aircraft flight dynamics model with all elevator jams that can be possibly mitigated using the available control authority. Section V serves as a conclusion.

II. PRELIMINARIES

A. The GTM Aircraft Flight Dynamics Model

The GTM aircraft dynamics model and the associated state variables and control inputs of the GTM aircraft flight dynamics are briefly described in the following. The model is represented by the state equation,

$$\begin{aligned} \dot{x}(t) &= f\left(x(t), u(t)\right) \\ x &= \left[\begin{array}{cccccccccccc} V & \alpha & \beta & p & q & r & pN & pE & h & \phi & \theta & \psi \end{array}\right]^T \\ u &= \left[\begin{array}{cccc} \delta_a & \delta_r & \delta_e & \delta_T \end{array}\right]^T \end{aligned} \tag{1}$$

The 12 state variables are V: total speed (knots), α: angle of attack (rad), β: side slip (rad), p: roll rate (rad/s), q: pitch rate (rad/s), r: yaw rate (rad/s), pN: position North (ft), pE: position East (ft), h: the altitude (ft), ϕ: roll angle (rad), θ: pitch angle (rad), and ψ: yaw (rad). The 4 control inputs are δ_a: aileron (rad), δ_r: rudder, (rad), δ_e: elevator (rad), and δ_T: throttle (%).

The aircraft flight dynamics state equation in (1) consists of 12 first-order differential equations, which are nonlinear, flight condition-dependent, and can be highly coupled. A common practice of flight control system design is to choose a flight trim associated with a desired flight condition, and obtain a linear 12-states model to represent the flight dynamics at and around the trim. Due to the symmetrical structure of the aircraft under most of the normal flight conditions, the linearized model can be further simplified by decoupling the model into two sets of state equations including the longitudinal and the lateral state equations.

For example, if the desired flight condition is a straight level flight with 5° angle of attack, then the corresponding trim will be determined by solving (1) as follows:

$$\begin{aligned} x_{trimA} &= \left[\begin{array}{ccccc} 75.13\text{kt} & 5^\circ & 0^\circ & 0^\circ/\text{s} & 0^\circ/\text{s} \end{array}\right. \\ &\quad \left.\begin{array}{ccccccc} 0^\circ/\text{s} & *\text{ft} & *\text{ft} & 800\text{ft} & -0.01^\circ & 5^\circ & *^\circ \end{array}\right]^T \\ u_{trimA} &= \left[\begin{array}{cccc} -0.01^\circ & 0.01^\circ & 0.97^\circ & 21.01\% \end{array}\right]^T \end{aligned} \tag{2}$$

The trim is called $Trim\ A$ for later reference. Note that the position state variables pN, pE, the altitude h and the yaw angle ψ are left unspecified since they are directly dependent on the other eight state variables and are irrelevant to the trim determination. It can also be seen on Fig. 1 that the values of Elevator, Throttle, Pitch, and Tas (Total air speed) at the angle of attack α=5° are consistent with $\delta_e = 0.968°$, $\delta_T = 21.01\%$, $\theta = 5°$ and V=75.13 knots. The $Trim\ A$ data in (2) and the GTM level flight trim graphs in Fig. 1 will be employed later in the design of control systems to enhance the stability and achieve the altitude and flight path tracking even when the elevator ceases to function and jams at a position, which is unknown $a\ priori$, in a wide range between $-4°$ and $4°$.

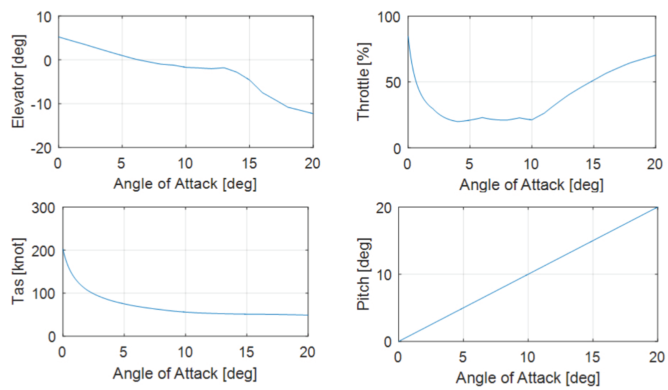

Fig. 1: GTM straight level flight trim graphs

B. Control Simulation Using the Full Nonlinear GTM Aircraft Flight Dynamics with Actuator Limits

The nonlinearities and uncertainties of the aircraft flight dynamics are considered in the control system design process. Furthermore, the full nonlinear GTM aircraft flight dynamics model developed by NASA is employed in the flight control simulations. The closed-loop control system

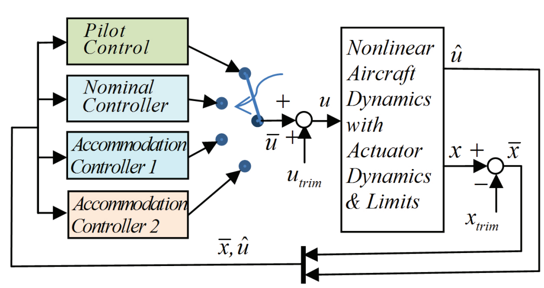

Fig. 2: Control simulation schematic diagram using the full nonlinear GTM aircraft flight dynamics.

simulation will be conducted based on the switching control schematic diagram shown in Fig. 2. In this diagram, we show four ways of closing the loop to control the flight of the aircraft. The first is the pilot control, which is a manual control with stability augmentation. The second is the nominal controller, which is designed to achieve stable flight path and altitude tracking for the healthy aircraft without actuator failures. These two approaches work fine if no failure occurs. But if the elevator jams, the original stability augmentation or the nominal controller will not work since the operating flight equilibrium has changed and the jammed elevator will create a persistent disturbance that may drive the aircraft into a loss-of-control situation.

The third is Accommodation Controller 1 and the fourth is Accommodation Controller 2, which are specifically designed to address the issues caused by the elevator jam that occurs anywhere in the range from $0°$ to $4°$ and from $-4°$ to $0°$, respectively. The design of the nominal controller and the accommodation controllers will be given in Section III. The explanations of why the nominal controller is inadequate and how the accommodation controllers work will also be given in Section III.

Note that the input and the output of the controllers are denoted as $\bar{x}$ and $\bar{u}$, respectively, and their relationships with x and u, respectively, are described by $\bar{x} = x - x_{trim}$ and $\bar{u} = u - u_{trim}$. Recall that x_{trim} and u_{trim} are the state vector and the control input vector of the aircraft at the trim, respectively and x and u are the untrimmed state vector and control input vector of the aircraft, respectively. In real practice or in the simulation according to the schematic diagram shown in Fig. 2, u does not connect to the aircraft dynamics model directly. Instead, u applies to the input of the actuator while the output $\hat{u}$ of the actuator is connected to the input of the aircraft dynamics model. The information of $\hat{u}$ including the elevator jam position are also fed back to the controllers.

The GTM actuator amplitude and rate limits are given as follows:

$$\begin{aligned}&-20° < \delta_a < 20°, -30° < \delta_r < 30°, -30° < \delta_e < 20°\\&0\% < \delta_T < 100\%, -300°/s < \dot{\delta}_a < 300°/s,\\&-300°/s < \dot{\delta}_r < 300°/s, -300°/s < \dot{\delta}_e < 300°/s\end{aligned} \quad (3)$$

and the four actuator dynamics are all identical with the transfer function given by $10\pi/(s + 10\pi)$.

III. Control System Design

As depicted in Fig. 2, the switching control scheme consists of four switching positions which are to be selected according to the functionality condition of the elevator control actuator. The stability augmentation control design used in the Pilot Control option is fairly standard [16], [17], and therefore is omitted in the following discussion. The Nominal Controller is designed to provide a stable straight level flight with flight path and altitude tracking capabilities for the GTM aircraft when the elevator control actuator is functioning normally. The Accommodation Controllers 1 and 2 are designed to achieve the same objective as the nominal controller when the elevator control actuator fails and jams at any position in the regions from $0°$ to $4°$ and from $-4°$ to $0°$, respectively.

A. Nominal Flight Path and Altitude Tracking Controller

For the trim of a straight level flight with $5°$ angle of attack, given in (2) as Trim A, a linearized state-space model with decoupled longitudinal and lateral dynamics state equations can be obtained via the standard Jacobian approach as follows,

$$\begin{aligned}&\dot{\bar{x}}_{Lg}(t) = A_{Lg}\bar{x}_{Lg}(t) + B_{Lg}\bar{u}_{Lg}(t)\\&\bar{x}_{Lg} = \begin{bmatrix} \bar{V} & \bar{\alpha} & \bar{q} & \bar{\theta} \end{bmatrix}^T, \ \bar{u}_{Lg} = \begin{bmatrix} \bar{\delta}_e & \bar{\delta}_T \end{bmatrix}\end{aligned} \quad (4)$$

$$\begin{aligned}&\dot{\bar{x}}_{La}(t) = A_{La}\bar{x}_{La}(t) + B_{La}\bar{u}_{La}(t)\\&\bar{x}_{La} = \begin{bmatrix} \bar{\beta} & \bar{p} & \bar{r} & \bar{\phi} \end{bmatrix}^T, \ \ u_{La} = \begin{bmatrix} \bar{\delta}_a & \bar{\delta}_r \end{bmatrix}^T\end{aligned} \quad (5)$$

The trim is stable although both of the longitudinal and lateral dynamics have poor characteristics: a very small longitudinal damping ratio $\varsigma = 0.04$ with slow natural frequency $\omega_n = 0.31\text{rad/s}$, and a small lateral damping ratio $\varsigma = 0.15$ with $\omega_n = 6\text{rad/s}$. Furthermore, since one of the lateral eigenvalues is -0.046 the system will take a long time (time constant = 21.7 seconds) to follow a step command or dissipate step disturbances.

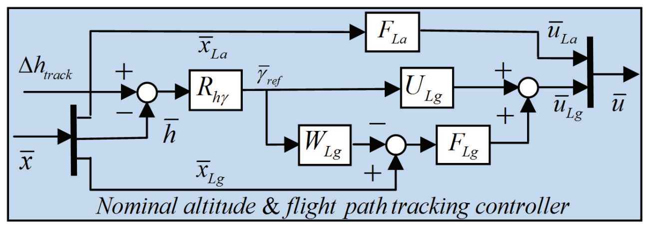

Fig. 3: Nominal stabilizing controller with altitude and flight path tracking capability for a healthy aircraft without actuator failures.

The nominal controller shown in Figure 3 is designed specifically for $Trim\ A$ to achieve a stable straight level flight with altitude and flight path tracking capabilities under the condition that all control input actuators are functioning normally. The longitudinal and lateral state feedback gain matrices F_{Lg} and F_{La} are computed using the LQR (Linear Quadratic Regulation) [16], [17], [18], [19] approach based on the state equations (4) and (5), respectively, as follows,

$$F_{Lg} = \begin{bmatrix} -0.2487 & -13.88 & 1.302 & 17.57 \\ -0.0836 & -0.058 & 0.0746 & 0.5518 \end{bmatrix}, \quad (6a)$$

$$F_{La} = \begin{bmatrix} -74.82 & 25.74 & 6.832 & 31.06 \\ -34.91 & -4.856 & 15.21 & -2.451 \end{bmatrix}. \quad (6b)$$

The longitudinal closed-loop damping ratio and natural frequency have been improved tremendously to $\varsigma = 0.645$, $\omega_n = 0.62\text{rad/s}$. The lateral closed-loop damping ratio and natural frequency are also better now $\varsigma = 0.62$ and $\omega_n = 6.71\text{rad/s}$. The lateral time constant associated with step disturbances is reduced to about 1 second from 21.7 seconds.

To achieve flight path tracking, the matrices U_{Lg} and W_{Lg} are obtained as

$$\begin{aligned}&W_{Lg} = \begin{bmatrix} 75.126 & -0.14905 & 0 & 0.85095 \end{bmatrix}^T\\&U_{Lg} = \begin{bmatrix} 12.844 & 450.19 \end{bmatrix}^T\end{aligned} \quad (7a)$$

from the following regulator equations [20], [21], [22], [23]:

$$A_{Lg}W_{Lg} + B_{Lg}U_{Lg} = 0, \quad C_{1u}W_{Lg} + D_{11u}U_{Lg} = 0 \tag{7b}$$

where A_{Lg} and B_{Lg} are given in (4), and $D_{11u} = 1$ and $C_{1u} = \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix}$ are selected to achieve flight path γ tracking. The h-γ gain constant $R_{h\gamma}$ is employed to achieve altitude regulation via flight path tracking. The gain $R_{h\gamma}$ is selected to have a reasonably fast tracking rate while not to cause large oscillations or actuator saturation.

B. Accommodation Controller 1 to Achieve Flight Path and Altitude Tracking for the Impaired Aircraft with Elevator Jam at Any Position Between 0° and 4°

This is a much more challenging control system design problem than the design of a nominal controller. The accommodation controller needs to stabilize and achieve flight path and altitude tracking for the impaired aircraft with an arbitrary elevator jam within the feasible level flight trim region. If an elevator jam occurs at a non-neutral position during the flight, the control system would need to address three arising critical issues immediately. They are the disappearance of the nominal flight equilibrium, the persistent disturbance created by the aerodynamic force on the jammed elevator control surface, and the loss of the elevator control authority.

Recently in [15], we developed a new approach to design a fixed controller that can stabilize the impaired aircraft and automatically lead the aircraft to a level flight equilibrium determined by any elevator jam position within the feasible level flight trim region. In this paper, we further improve the altitude tracking accuracy and extend the covering range of elevator jam positions by incorporating a nonlinear parameter adaptation involving $\Delta h_{desired}$, Δh_{track}, and δ_{e_trim} in the accommodation control feedback loop.

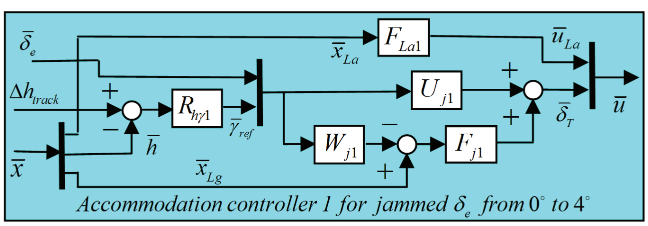

Fig. 4: Accommodation controller 1 to stabilize and achieve flight path and altitude tracking for the impaired aircraft with elevator jammed at anywhere within the range of 0 to 4 degrees.

The proposed approach in this paper does not require the elevator jam position $a\ priori$. It also does not need the level flight trim associated with the elevator jam position. Yet, a single fixed linear servomechanism-based controller with a nonlinear parameter adaptation is able to stabilize and achieve flight path and altitude tracking for the impaired aircraft with elevator jammed at anywhere within the range of 0° to 4°.

The block diagram of the Accommodation Controller 1 is shown in Fig. 4. Since the trim at which the impaired aircraft will fly is unknown $a\ priori$, we will select a straight level flight with an angle of attack close to the possible trim that the impaired aircraft would fly. The elevator jam region between 0° and 4° covered by Accommodation Controller 1 is associated with the range of the angle of attack less than 6° according to the level flight trim graphs shown in Fig. 1. The Trim A described in (2) is a good candidate since it is a straight level flight with 5° angle of attack. Therefore the longitudinal and lateral state equations will be the same as those in (4) and (5).

The design of the lateral state feedback gain matrix F_{La1} remains the same as that used for the nominal controller. But the longitudinal controller is required to be designed to address the elevator jam and flight path tracking according to the new longitudinal state equation,

$$\dot{\bar{x}}_{Lg}(t) = A_{Lg}\bar{x}_{Lg}(t) + B_{j1}\bar{\delta}_T(t) + [B_{e1}\ 0]\begin{bmatrix} \bar{\delta}_e \\ \bar{\gamma}_{ref} \end{bmatrix} \tag{8}$$

where $[B_{e1}\ B_{j1}] = B_{Lg}$ and $\bar{\delta}_e$ becomes a persistent disturbance, not a control input anymore. Using the LQR approach, the state feedback gain matrix F_{j1} is designed as

$$F_{j1} = \begin{bmatrix} -1.7542 & 206 & -12.42 & -230.3 \end{bmatrix} \tag{9}$$

With the above F_{j1}, the longitudinal closed-loop will have damping ratio and natural frequency $\varsigma = 0.387$ and $\omega_n = 0.335\mathrm{rad/s}$. As expected, the stability is not as good as that of the healthy aircraft. The tracking regulator matrices W_{j1} and U_{j1} can be obtained as

$$W_{j1} = \begin{bmatrix} 12.864 & -90.1 \\ -0.0214 & 0.126 \\ 0 & 0 \\ -0.0214 & 1.126 \end{bmatrix}, U_{j1} = \begin{bmatrix} -3.998 & 501.55 \end{bmatrix} \tag{10}$$

by solving the following regulator equations,

$$\begin{array}{l} A_{Lg1}W_{j1} + \begin{bmatrix} B_{e1} & 0 \end{bmatrix} + B_{j1}U_{j1} = 0 \\ C_{1uj1}W_{j1} + \begin{bmatrix} 0 & D_{11uj1} \end{bmatrix} = 0 \end{array} \tag{11}$$

with $C_{1uj1} = [0\ 1\ 0\ -1]$ and D_{11uj1}=1. Since the impaired aircraft is less stable, the h-γ gain is chosen to be smaller as $R_{h\gamma 1} = R_{h\gamma}/5$ in the accommodation controller.

As mentioned earlier, the accommodation controller is designed to perform three tasks: (a) stabilize the aircraft, (b) maintain a level flight at steady state, and (c) perform an ascent or descent operation whenever needed. Unlike the nominal controller that stabilizes the healthy aircraft to the original trim, Trim A, the Accommodation Controller 1 will stabilize the impaired aircraft at an equilibrium determined by the elevator jam position and the straight level flight condition. Note that the original trim has disappeared, and the only possible straight level flight trim is determined by the elevator jam position, which is unknown $a\ priori$. The accommodation controller is pre-designed to automatically reach any level flight trim determined by the elevator jam position. As long as the elevator jam position is not out of range, the linear, fixed accommodation controller shown in Fig. 4 will guarantee the stability at the new straight level flight equilibrium and be capable of performing ascent and descent maneuvers.

Unlike the nominal controller, Δh_{track} will not be equal to $\Delta h_{desired}$ when there is an elevator jam unless the jam is at the neutral position. A nonlinear parameter adaptation procedure is created to construct a lookup table that describes the relationship among Δh_{track}, $\Delta h_{desired}$, and δ_{e_trim} as $\Delta h_{track} = g_1(\Delta h_{desired}, \delta_{e\_jam})$, where $\Delta h_{desired}$ is the desired change of the altitude from the nominal altitude and δ_{e_trim} is the jam position of the elevator. The work of the accommodation controller will be demonstrated in Section IV.

C. Accommodation Controller 2 to Achieve Flight Path and Altitude Tracking for the Impaired Aircraft with Elevator Jam at Any Position Between $-4°$ and $0°$

The control system structure and design methodology employed in Accommodation Controller 2 are identical to that in Accommodation Controller 1 of the previous subsection, except that Accommodation Controller 2 is specifically designed to deal with another region of elevator jam. In case that the elevator jam occurs in the region between $-4°$ and $0°$, the corresponding angle of attack will be much higher than those considered in the design of Accommodation Controller 1 according to the level flight trim graphs shown in Fig. 1. Therefore, we will select a straight level flight trim with 15° angle of attack, denoted by $Trim\ B$, in the design of Accommodation Controller 2.

$$\begin{aligned} x_{trimB} = \big[& \ 51.23\text{kts} \quad 15° \quad 0° \quad 0°/\text{s} \quad 0°/\text{s} \\ & 0°/\text{s} \quad *\text{ft} \quad *\text{ft} \quad *\text{ft} \quad -0.01° \quad 15° \quad *° \ \big]^T \\ u_{trimB} = \big[& -0.11° \quad 0.05° \quad -4.61° \quad 51.39\% \ \big] \end{aligned} \tag{12}$$

The design procedure of Accommodation Controller 2 based on $Trim\ B$ is the same as Accommodation Controller 1, and therefore is omitted. The work of the accommodation controller will be demonstrated in Section IV.

IV. SIMULATIONS

The results of 8 simulations will be presented in this section and displayed in the two figures: Fig. 5 and Fig. 6, where each figure will have two columns of graphs and each column will consist of two simulations: one in solid lines and the other in dotted lines. The solid-line will be referred as the a-simulations and dotted-line as the b-simulations. The columns will be numbered from 1 to 4 starting from the left column of Fig. 5. Therefore the solid-line simulation on the left column of Fig. 6 will be referred as Case 3a simulation. In all the simulations, the initial state is assumed at $Trim\ A$ with the two position states pN=0ft, pE=0ft, the altitude h=800ft, and the yaw angle $\psi = 90°$.

The nominal controller designed in Section IIIA is employed in the Case 1a and 1b simulations, where the aircraft is assumed healthy without any actuator failures. The altitude tracking inputs Δh_{track} = $\Delta h_{desired}$ = +200ft and Δh_{track} = $\Delta h_{desired}$ = -200ft are applied in Cases 1a and 1b to command the aircraft to ascend to 1000ft and descend to 600ft, respectively. The system will reach a steady state as $t \to \infty$. Note that the steady state is $Trim\ A$.

In Cases 2a and 2b, elevator jams were assumed to occur at t=20s, but no action was taken to mitigate the failure and the nominal controller, now inadequate, still continued

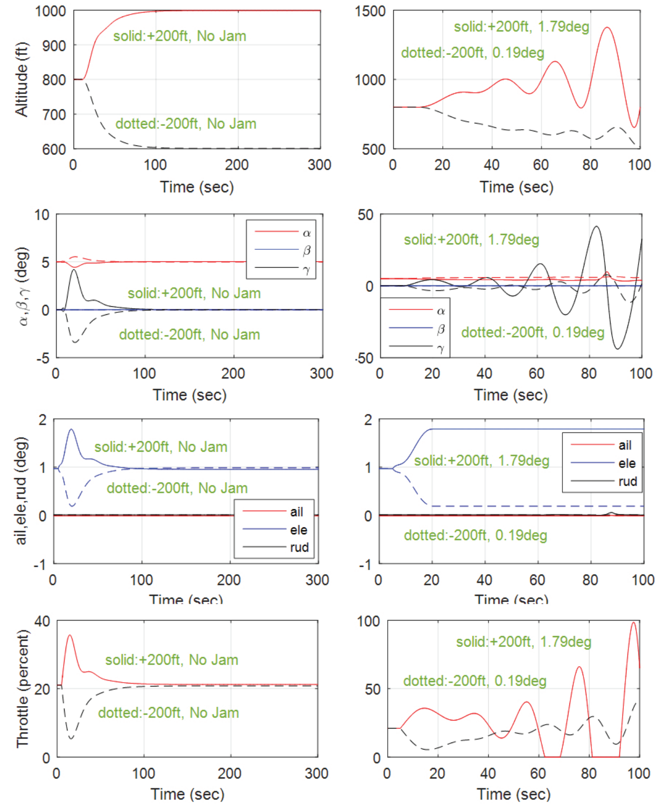

Fig. 5: Case 1a (solid) and 1b (dotted) on the left column, Case 2a (solid) and 2b (dotted) on the right column.

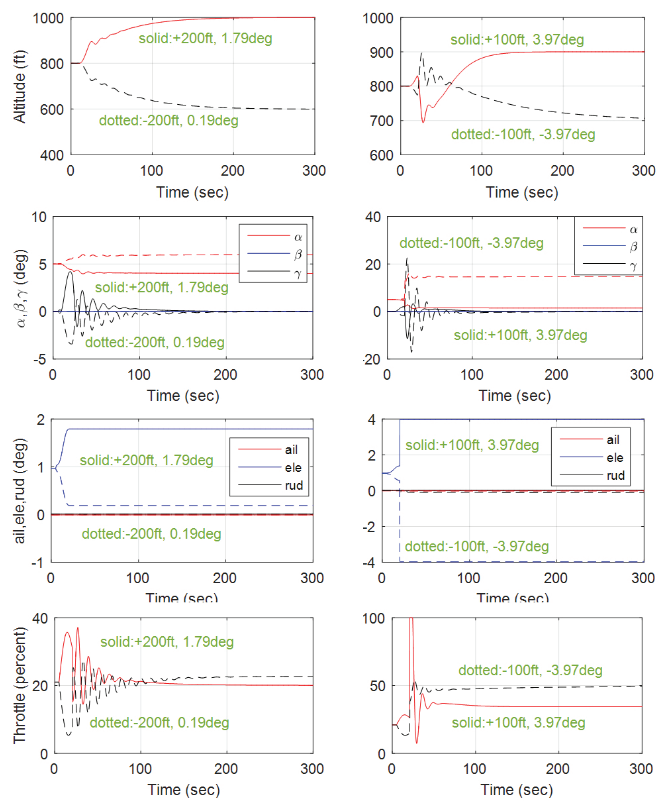

Fig. 6: Case 3a (solid) and 3b (dotted) on the left column, Case 4a (solid) and 4b (dotted) on the right column.

its old control strategy without knowing the loss of the elevator control, the presence of the newly created persistent disturbance, and the disappearance of the original trim. It can be seen from the graphs on the right column of Fig. 5 that the aircraft lost stability shortly after the elevator jam for both Cases 2a and 2b.

Similar to Case 2a, the elevator was stuck at $\delta_{e\_jam} = 1.79°$ during the $\Delta h_{desired}$=+200ft ascent at t=20s, but in Case 3a the Accommodation Controller 1 was engaged at t=21s to mitigate the effect of the elevator jam failure. Δh_{track}=141.6ft was applied as the altitude tracking input in the Accommodation Controller 1 to achieve the altitude tracking. Similarly in Case 3b, Δh_{track}=-96.9ft was used in Accommodation Controller 1 to complete the desired altitude decent under the elevator $\delta_{e\_trim} = 0.19°$ jam condition. Recall that Δh_{track} is a function of $\Delta h_{desired}$ and δ_{e_trim} and can be obtained from a lookup table representing the function.

Since the system is stable, it will reach a steady state as $t \to \infty$. The state and control input at t=500s for Cases 3a and 3b are found as follows,

$$\begin{aligned}
x_{3a}(500) &= [82.34\text{kt} \;\; 4.01° \;\; 0° \;\; 0°/s \;\; 0°/s \\
&\quad 0°/s \;\; *\text{ft} \;\; *\text{ft} \;\; 1000.1\text{ft} \;\; 0.02° \;\; 4.01° \;\; *°]^T \\
u_{3a}(500) &= [-0.01° \;\; 0.01° \;\; 1.79° \;\; 20.07\%]^T \\
x_{3b}(500) &= [69.49\text{kt} \;\; 5.97° \;\; 0° \;\; 0°/s \;\; 0°/s \\
&\quad 0°/s \;\; *\text{ft} \;\; *\text{ft} \;\; 598.9\text{ft} \;\; -0.03° \;\; 5.97° \;\; *°]^T \\
u_{3b}(500) &= [-0.01° \;\; 0.01° \;\; 0.19° \;\; 22.75\%]^T
\end{aligned}$$

Note that these are straight level flight equilibriums with different angles of attack. As long as the controller is designed to reach a stable level flight at steady state, the aircraft with the help of the controller will automatically fly to the specific equilibrium determined by the elevator jam position.

In Case 4a, the Accommodation Controller 1 with Δh_{track}=661.1ft is employed to mitigate the failure caused by the elevator jam at $\delta_{e\_trim} = 3.97°$ and achieve the desired altitude $\Delta h_{desired}$=100ft ascent. Similarly in Case 4b, the Accommodation Controller 2 with Δh_{track}=-228.9ft is employed to mitigate the failure caused by the elevator jam at $\delta_{e\_trim} = -3.97°$ and achieve the desired altitude $\Delta h_{desired}$=-100ft descent.

Even at the extreme flight conditions, the system is still stabilized and will reach a steady state as $t \to \infty$. The state and control input at t=500s for Cases 4a and 4b are found as follows,

$$\begin{aligned}
x_{4a}(500) &= [118.56\text{kt} \;\; 1.48° \;\; 0° \;\; 0°/s \;\; 0°/s \\
&\quad 0°/s \;\; *\text{ft} \;\; *\text{ft} \;\; 900.4\text{ft} \;\; 0.01° \;\; 1.48° \;\; *°]^T \\
u_{4a}(500) &= [0° \;\; 0.01° \;\; 3.97° \;\; 34.36\%]^T \\
x_{4b}(500) &= [51.31\text{kt} \;\; 14.64° \;\; -0.03° \;\; 0°/s \;\; 0°/s \\
&\quad 0°/s \;\; *\text{ft} \;\; *\text{ft} \;\; 699.8\text{ft} \;\; -0.16° \;\; 14.64° \;\; *°]^T \\
u_{4b}(500) &= [-0.08° \;\; 0.02° \;\; -3.97° \;\; 49.35\%]
\end{aligned}$$

Cases 4a and 4b are operating at extreme flight conditions. Due to the elevator jam at $\delta_{e\_trim} = 3.97°$, Case 4a is forced to fly at $\alpha = 1.48°$, a low angle of attack with low lift coefficient, in order to keep a stable level flight. On the other hand, in Case 4b the elevator jam at $\delta_{e\_trim} = -3.97°$ only allowed the aircraft to fly at $\alpha = 14.64°$, a high angle of attack with near-stall low speed, in order to stay aloft at a level flight. Nevertheless, the impaired aircraft is still able to reach a perfect straight level flight equilibrium in both extreme cases.

V. Conclusion

Two accommodation controllers have successfully addressed the critical aircraft flight safety issues caused by the elevator jam in two regions: one from $0°$ to $4°$, and the other from $-4°$ to $0°$. These two regions together cover almost all feasible level flight trims. If the elevator jam position is outside these two regions, there would be either no level flight trim available or the trim would be near stall and very difficult to fly.

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